DOCUMENT RESUME

ED 309 063 SE 050 716

TITLE	Survey of Academic Skills: Grade 12 Rationale and Content Mathematics.
INSTITUTION	California State Dept. of Education, Sacramento. Bureau of Publications.
REPORT NO	ISBN-0-8011-0808-X
PUB DATE	89
NOTE	88p.
AVAILABLE FROM	Bureau of Publications, Sales Unit, California State Department of Education, P.O. Box 271, Sacramento, CA 95802-0271 ($2.50 plus sales tax for California residents).
PUB TYPE	Tests/Evaluation Instruments (160)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Grade 12; High Schools; Mathematical Concepts; Mathematics Achievement; Mathematics Materials; *Mathematics Skills; Mathematics Tests; *Secondary School Mathematics; *Test Construction; Testing Programs
IDENTIFIERS	*California

ABSTRACT

This survey reflects new developments in the mathematics curriculum as emphasized in the curriculum frameworks and state-adopted textbooks. The first section describes the procedure for the development of the survey and the rationale for the topics and types of questions included on the mathematics portion of the test. The second section describes the mathematics content to be assessed and includes illustrative test items. Categories assessed are: (1) "Problem Solving/Reasoning"; and (2) "Understandings and Applications." An answer key to the examples is provided. (YP)

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California Assessment Program

Survey of Academic Skills: Grade 12

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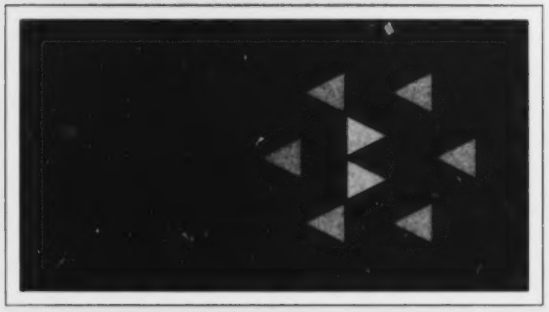
Rationale and Content

Mathematics



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Survey of Academic Skills: Grade 12

Rationale and Content

Mathematics

Publishing Information

The *Survey of Academic Skills: Grade 12, Rationale and Content, Mathematics* was compiled by the California Assessment Program, California State Department of Education, 721 Capitol Mall, Sacramento, California (mailing address: P.O. Box 944272, Sacramento, CA 94244-2720).

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ISBN 0-8011-0808-X

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California Assessment Program

Survey of Academic Skills: Grade 12 Mathematics

Introduction

Senate Bill 1889 of 1984 required that the California Assessment Program's test, the *Survey of Basic Skills*, be revised and broadened to reflect the areas of emphasis in the *Model Curriculum Standards, Grades Nine Through Twelve* and the newly adopted curriculum frameworks. The important steps of the test development process are highlighted below:

- All California districts with high schools were invited to participate in the development of the new test.
- An assessment advisory committee for mathematics drafted preliminary test content specifications for test items based on the *Model Curriculum Standards* and state curriculum frameworks. The advisory committee was composed of curriculum specialists from the following groups: school districts, offices of county superintendents of schools, professional associations, colleges, The California State University, the University of California, and the State Department of Education. Names of the statewide committee members, ad hoc committee members, and special consultants are listed on pages ix–x.
- CAP staff, with the help of the assessment advisory committee, created preliminary test content specifications and sample test items. Using the sample items as models, a group of item writers developed items for each of the proposed skills.
- Departmental staff reviewed the test items for alignment with the proposed test content specifications and the *Model Curriculum Standards*. The assessment advisory committee and CAP staff reviewed, screened, and revised the preliminary item pool and prepared it for field testing.
- In 1986 and 1987, items were field-tested in more than 300 school districts on approximately 20,000 students. These tryouts allowed CAP to assess the difficulty of each test item, the clarity of directions, and such problems as bias, unclear wording of items, inappropriate response choices, or unfamiliar formats among the items so that only the best items would survive the analysis of the field test.

- The assessment advisory committee reviewed the results of both field tests and, working with the CAP staff, selected the final set of 300 items. The final items were then reviewed again by CAP staff and testing professionals. The items were reviewed for linguistic, ethnic, and gender bias. In addition, a variety of item statistics were examined in search of otherwise undetected defects and sources of bias.

Rationale

The *Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve* (1985) emphasizes the need to raise expectations for all students graduating from high school:

> To enable all graduates to meet current and future demands, mathematics education must focus on students' capacity to make use of what they have learned in all settings. Mathematical power, which involves the ability to discern mathematical relationships, reason logically, and use mathematical techniques effectively, must be the central concern of mathematics education and must be the context in which skills are developed. (p. 1)

The California Assessment Program (CAP) has rewritten the grade twelve test in order to align it with the *Mathematics Framework for California Public Schools* and the *Model Curriculum Standards*. The major difference between the revised CAP test—now called *Survey of Academic Skills, Grade 12*—and the older version of the *Survey* is that the new test emphasizes understanding of mathematical concepts and problem solving. The new grade twelve *Survey* is designed to measure what students understand about the mathematical concepts and skills they have learned in kindergarten through grade twelve and how well they can use the mathematics they have learned in familiar and unfamiliar problem situations. This test will also assess the students' abilities to estimate, discern relationships, and use number sense in the evaluation and interpretation of intermediate and final results of a problem-solving process. It requires students to use higher-level thinking skills and therefore provides a measure of their ability to do so in a mathematical setting as opposed to a measure only of their ability to perform rote mathematical algorithms that may be correctly done but not well understood.

The major reporting categories are as follows:

I. **Problem Solving/Reasoning**
   - A. Problem Formulation
   - B. Analysis and Strategies
   - C. Interpretation of Solutions
   - D. Nonroutine Problems/Synthesis of Routine Applications

II. **Understandings and Applications**
   - A. Numbers and Operations
   - B. Patterns, Functions, and Algebra
   - C. Data Organization and Interpretation/Probability
   - D. Measurement, Geometry, and Spatial Relationships
   - E. Logical Reasoning

Besides receiving an overall mathematics score, each school will receive a score for problem solving and a score for understandings and applications. These two major categories will be subdivided into four and five categories, respec-

tively, as shown previously. Each of the five categories for understandings and applications will be further subdivided as shown on page xi. For example, Numbers and Operations will have two subscores: (1) nature of real numbers; and (2) selection and use of operations on real numbers. The intent of this reporting scheme is to enable teachers, schools, and school districts to judge the effectiveness of their mathematical programs in teaching mathematical understanding and the use of mathematical concepts and skills in applied problem-solving situations. Situational lessons that encompass several mathematical content areas and give rise to several problems exemplify this approach by encouraging students to be creative, ask questions, verify and interpret results, and develop different approaches for finding solutions. This situational approach contrasts sharply to narrow exercises that provide practice with one specific skill.

In the reporting scheme discussed previously, problem solving is listed as a major reporting category in itself; however, it is actually present in all categories. In order to prepare students adequately, the school's mathematics program must present students with problems that use acquired skills and understanding of problem-solving strategies. Synthesis and evaluation are higher levels of thinking that must be systematically developed; problems with too much or too little information, or even with inconsistent data, should be included to encourage this level of thinking.

The writing committee used the *Mathematics Framework* and *Model Curriculum Standards (MCS)* as guidelines in developing new questions. Although the *Framework* and *MCS* list topics such as geometry and measurement separately, in CAP test development and reporting such related topics are combined. The committee felt that this scheme of reporting will encourage teachers to embed related topics in the same instructional module.

A word about calculators and their use during testing is important. The use of calculators is encouraged in the *Mathematics Framework*, and students need to know when and how they are best utilized. Even though the committee realized that the use of calculators as an instructional tool is controversial, the committee decided to allow their use on the twelfth grade test because:

- Calculators and computers are common, everyday tools in many jobs.
- Calculators are convenient and provide accurate square root, logarithmic, and trigonometric values.
- Calculators permit exploration and investigation of new algorithms.
- Calculators can allow more instructional time for developing understanding of concepts and solving routine and nonroutine problems.

This document should not be used as a teaching guide, because many classroom goals and activities in the *Mathematics Framework* do not lend themselves to being measured by multiple-choice questions. For example, making geometric constructions and using models and manipulatives are all important learning experiences that help the student progress from the concrete to the abstract. However, this type of learning is not easily measured.

It should be recognized that the following list of content specifications represents a substantial increase in mathematical understanding expected of *all* students by grade twelve. Although these increased expectations reflect the *Mathematics Framework* and the *Model Curriculum Standards*, some schools may experience a two- to three-year transition period before their curriculums and instructional programs adequately prepare all of their students for this assessment. For those schools the assessment will serve as a monitoring tool as the gradual implementation takes place.

Resource Materials

As described in the rationale, the content and approach of the twelfth grade *Survey* is based on the assumption that students have completed mathematics courses equivalent to Math A and Math B cited in the *Mathematics Framework for California Public Schools*. At this writing there are no textbooks that incorporate the mathematical ideas and problem-solving contexts that match the specifications for Math A and B. However, during the past several years, a number of school districts or consortia have developed their own instructional materials. The Department of Education is consolidating and extending materials for a Math A course, and it plans to publish a draft of the material by August, 1989, for field-testing in September, 1989. For additional information about the pilot project, write to: Math A Project, Math/Science Unit, California State Department of Education, 721 Capitol Mall, P.O. Box 944272, Sacramento, CA 94244-2720; or call (916) 324-7190.

Mathematics Assessment Advisory Committee

Janet Abbott
Chula Vista City School District

Joan Akers
San Diego County Office of Education

James Caballero
UCLA

Joe Cooney
National Alliance of Business

Clyde Corcoran, Committee Chair
Whittier Union High School District

Richard Dean
California Institute of Technology

Sister Rose Eleanor Ehret
Holy Names College

Joan Gell
Palos Verdes Peninsula Unified School District

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Vice Chair

Bob Hamada
Los Angeles Unified School District

Joe Hoffmann
State Department of Education

Roberta Koss
Tamalpais Union High School District

Thomas J. Lester
San Juan Unified School District

Gail Lowe
Conejo Valley Unified School District

Sandra Marshall
San Diego State University

Vance Mills
San Diego City Unified School District

Susan Ostergard
University of California, Davis

Henry Palmer
Los Angeles County Education Center

Annie Podesto
Stockton City Unified School District

Gail Robinette
Fresno Unified School District

Ed Silver
San Diego State University

Tony Spears
Fresno County Office of Education

Jean Stenmark
University of California, Berkeley, EQUALS

Harold D. Taylor
San Mateo Union High School District

Shirley Trembley
Bakersfield College

Lee Webb
California State University, Bakersfield

Les Winters
Los Angeles Unified School District

Dorothy Wood
Tamalpais Union High School District

Tej Pandey, State Department of Education, Consultant to the Committee

Special Advisers

Robert Blackburn
Los Angeles Unified School District

Charles Clark
Salinas Union High School District

Carrol Dahl
Fresno Unified School District

Dorothy Dahl
Fresno Unified School District

Carol Dean
Pasadena Area Community College

Floyd Downs
Arizona State University

Linda DuCray
Hartnell College

Don Gernes
El Dorado Union High School District

Mary Kantowski
University of Florida

Kathleen Kellebrew
Sonoma Valley Unified School District

Diane Krantz
Consultant in Mathematics Education

Clarita Montalban
Jurupa Unified School District

Jerry Neumann
Garden Grove Unified School District

Laura Robinson
Irvine Unified School District

Tom Romberg
University of Wisconsin

James Rudolph
Chaffey Joint Union High School District

George van Buskirk
Campbell Union High School District

Don Volle
Paso Robles Joint Union High School District

Brandon Wheeler
Sacramento City College

Jim Wilson
University of Georgia

Reporting Categories
Survey of Academic Skills: Grade 12
Mathematics

I. Problem Solving/Reasoning

A. Problem Formulation
B. Analysis and Strategies
C. Interpretation of Solutions
D. Nonroutine Problems/Synthesis of Routine Applications

II. Understandings and Applications

A. Numbers and Operations
   1. Nature of Real Numbers
   2. Selection and Use of Operations on Real Numbers

B. Patterns, Functions, and Algebra
   1. Patterns
   2. Relations, Functions, and Graphs
   3. Algebra

C. Data Organization and Interpretation/ Probability
   1. Organizing Data as Graphs and Charts
   2. Statistics
   3. Probability and Systematic Counting

D. Measurement, Geometry, and Spatial Relationships
   1. Mensuration
   2. Geometric and Spatial Relationships

E. Logical Reasoning
   Quantifiers, Connectives, Relationships, and Using Deductive and Inductive Reasoning

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
I. Problem Solving/ Reasoning			
A. Problem Formulation	12	The student will: • identify relevant mathematical questions or problems that would arise from a description of a situation • identify reasonable conjectures that could arise from a given problem situation • select problem situations that could be represented by given mathematical models—number sentences, equations, graphs, tables, diagrams, and so forth (Example 1) • identify data or information needed to formulate a useful mathematical problem that is related to the description of a given situation (Example 2)	*Example 1.* Which of the following problems can be solved by using the equation $x + 2 = 28$? o A math class started with 28 students. The next day 2 more students enrolled in the class. How many students does this class have now? o Erin added 2 more books to her collection. If she now has 28 books, how many books did Erin have originally? o Tim had $28 in his account. A week later he deposited $2 more. How much money does he have in his account now? o Ann biked 28 km at 2 km per hour. How long did Ann bike?

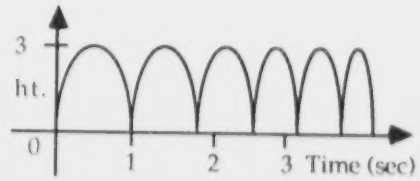
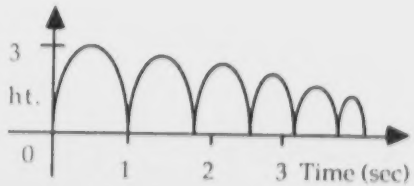
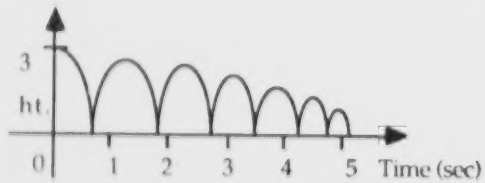
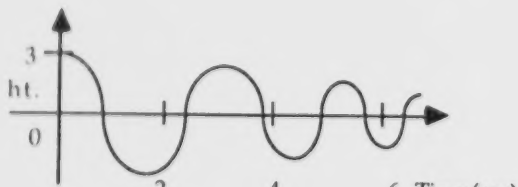
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
Problem Formulation (cont'd.)			*Example 2.* In the school parking lot there are 58 white cars, 17 red cars, 43 blue cars, 39 other colored cars, and 37 empty parking spaces. There are 415 students in school. To find the probability of a space selected at random of having a blue car, you would need to know which facts? o Number of blue cars and number of nonwhite cars o Number of blue cars and number of students o Number of blue cars and total number of cars in the lot o Number of blue cars and number of spaces in the parking lot

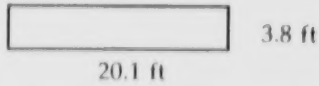
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
B. Analysis and Strategies	12	The student will: • restate a given problem in an alternate form (Example 3) • identify unknowns and/or given information • identify purpose or goal of a given problem • identify missing and/or extraneous information in a given problem • use guess and check or estimations to predict reasonable solutions and/or select procedures for finding solutions • identify underlying arithmetical, geometric, algebraic, probabilistic, or logical patterns or relationships in a given problem (Example 4) • identify simpler problems that could lead to the solution of a given problem	*Example 3.* On Monday-night football, the Los Angeles Raiders' record was 17 wins and 3 losses. Which statement gives the same information in a different way? o The Raiders won 17 games and lost 3 during the season. o The Raiders lost 17 of 20 games they played on Monday nights. o The Raiders' odds of winning the next Monday-night game are 17 to 3. o The Raiders won 17 of 20 games they played on Monday night. *Example 4.* At the beginning of the day a soft drink machine is supplied with 50 cans of soda and 85 cents in change. Sodas cost 45 cents each. At the end of the day $15.25 is in the machine. Which of the following processes should be used to determine the number of sodas sold that day? o divide o multiply o subtract then multiply o subtract then divide

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
B. Analysis and Strategies (cont'd.)		• identify useful mathematical models of a given problem—diagrams, graphs, tables, geometric models, equations, and so forth (Example 5)	*Example 5.* Which graph best illustrates the height of a bouncing tennis ball which was dropped from a height of 3 feet above the ground? o  o  o  o 

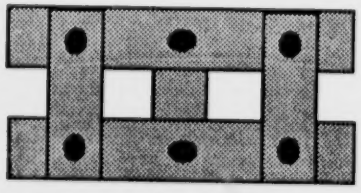
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
C. Interpretation of Solutions	12	The student will: • verify the solution. Was the problem solved? • recognize a sensible (plausible) solution (Example 6) • identify any reasonable conclusion or conjecture based upon the solution of a problem • identify simplifying assumptions made in the construction of a mathematical model used to solve a problem and their effect on the validity of the solution (Example 7) • identify similar problems that can be solved using the same or similar procedures that were used to solve another problem	*Example 6.* Find the area of the rectangle.  Sandra worked the problem on her calculator and got an answer of 7.638 square feet. She knew this was wrong because the number of square feet should be about: o $2 \times 4 = 8$ o $201 \times 4 = 804$ o $20 \times 4 = 80$ o $20 \times 3 = 60$ *Example 7.* Tina is told that two sides of Δ ABC are 3 cm each. Tina then performs the following operations to calculate the third side. $c^2 = 3^2 + 3^2$ $c^2 = 18$ $c = \sqrt{18}$ What must Tina have assumed for her solution? o Δ ABC is an acute triangle. o Δ ABC is an isosceles triangle. o Δ ABC is an obtuse triangle. o Δ ABC is an isosceles right triangle.

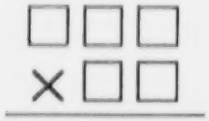
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
D. Nonroutine Problems/ Synthesis of Routine Applications	24	The student will: • solve problems involving both algorithmic and nonalgorithmic procedures, such as pattern recognition, inductive reasoning, extension of concepts, and simulation; e.g., geometric or numerical modeling or probabilistic situations (Examples 8, 9)	*Example 8.* A cardboard piece shaped as an equilateral triangle with each side 6 cm long is rolled to the right a number of times. If the triangle stops so that the letter "T" is again in the upright position, which one of the following distances could it have rolled? o 24 cm o 60 cm o 30 cm o 90 cm

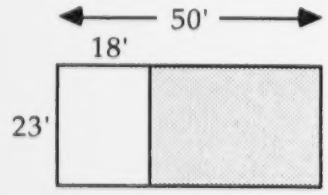
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
D. Nonroutine Problems (cont'd.)			*Example 9.* A machine shop has the following materials on hand: 60 long "three hole" boards 60 short "two hole" boards ● 60 fasteners What is the maximum number of structures shown below that can be built with the materials on hand?  o 10 o 20 o 12 o 30

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
II. Understandings and Applications A. Numbers and Operations 1. Nature of Real Numbers	12	The student will: • identify place value concepts in relation to largest or least, different numeration systems, and scientific notation (Example 10) • recognize order relationships between elements of different sets of numbers, including signed numbers, powers, number lines, and absolute values (Example 11) • estimate and/or judge reasonableness of a calculation (Example 12) • identify and/or use basic properties of real numbers; e.g., even, odd, multiples, and so forth • identify basic properties of the real number system; e.g., inverse, identity, distributive, and so forth (Example 13)	 *Example 10.* The five digits 1, 2, 3, 4, and 5 are placed in the boxes above to form a multiplication problem. If the digits are placed to give the maximum product, that product will fall between: o 10,000 and 22,000 o 22,001 and 22,300 o 22,301 and 22,400 o 22,401 and 22,500 *Example 11.* John is giving away $500. He gives 1/4 to Bill, 2/5 to Ken, and the rest to Mary. List the names according to the amount each receives (from most to least). o Mary, Ken, Bill o Bill, Ken, Mary o Ken, Mary, Bill o Mary, Bill, Ken

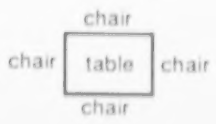
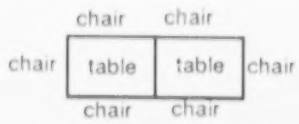
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
1. Nature of Real Numbers (cont'd.)			*Example 12.* The area of a square plot of land is 3/4 of a square mile. What is the approximate length of one side? o less than .8 mile o between .8 mile and .9 mile o between .9 mile and 1 mile o greater than 1 mile *Example 13.*  The dimensions of a school room are 50 feet by 23 feet. A partition is put up to form two separate rooms of dimensions shown above. How could the area, in square feet, of the larger room (shaded portion) be represented? o 23(50 – 18) o 50 – 18 • 23 o 50(23 – 18) o 50 • 23 – 18

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Selection and Use of Operations on Real Numbers	24	The student will: • recognize and use the six operations (addition, subtraction, multiplication, division, powers, and roots) in correct order within a problem situation (Examples 14, 15) • be able to use ratio, proportion, and percent (Example 16)	*Example 14.* A real estate agent purchased a house for $160,000. Then the agent sold the house for $170,000. Later, the agent purchased the house again for $180,000. Finally, the agent sold the house for $190,000. Which of these statements is true? o The agent came out even on the transactions. o The agent made $10,000. o The agent made $20,000. o The agent made $30,000. *Example 15.* On January 1, 1987, the tax on a house was $1000. If the taxes on the house increase by 10% each year, how much will the tax be on January 1, 1990? o $1100 o $1300 o $1210 o $1331 *Example 16.* Two organizations have memberships of 150 and 300. Together, they are to send 48 delegates to a convention, in proportion to their memberships. Which shows the number of delegates the 2 organizations should send? o 12 and 36 o 16 and 32 o 15 and 30 o 24 and 24

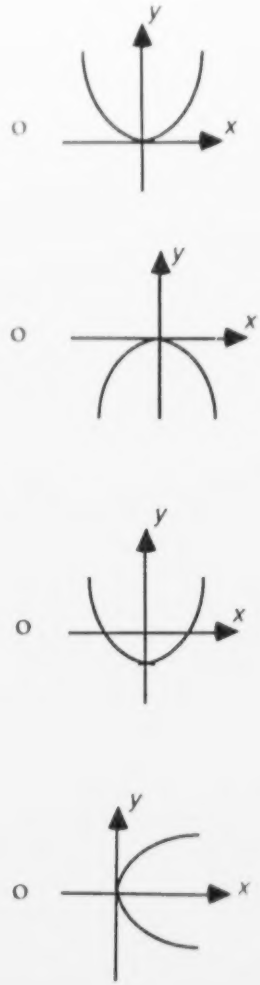
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
B. Patterns, Functions, and Algebra			
1. Patterns	12	The student will: • identify patterns in derived or given sequences of numbers (Example 17) • interpret relationships found in given or derived sets of data (Example 18)	*Example 17.* Figure 1  Figure 2  The tables at the pizza parlor can accommodate 4 chairs as shown in Figure 1. When 2 tables are put together, 6 people can be seated on the chairs as shown in Figure 2. How many tables are needed to make a long table seating 12 people? o 3 o 4 o 5 o 6

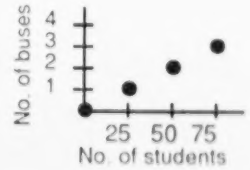
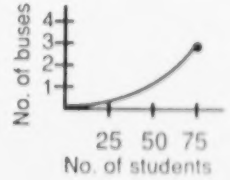
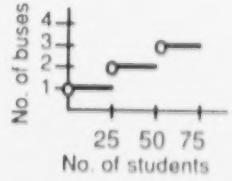
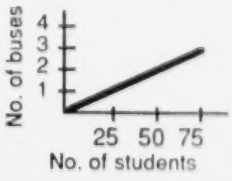
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
1. Patterns (cont'd.)			*Example 18.* For these sequences of consecutive integers ⇒ These are true statements 2, 3, 4 ⇒ $2 \cdot 4 = 8$ and $3^2 = 9$ 15, 16, 17 ⇒ $15 \cdot 17 = 255$ and $16^2 = 256$ 108, 109, 110 ⇒ $108 \cdot 110 = 11{,}880$ and $109^2 = 11{,}881$ Which of these could be a model of the relationship between the whole numbers as shown in the above statements? I. $(x)(x + 2) = (x + 1)^2 - 1$ II. $(x - 1)(x + 1) = x^2 - 1$ III. $(x)(x + 1) = (x + 2)^2 - 1$ o I and II only o I and III only o II and III only o I, II, and III

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Relations, Functions, and Graphs	12	The student will: • identify and/or use graphs, tables, or diagrams that represent functions (Examples 19, 20) • use direct and inverse variation and inverse functions (Example 21) • identify and/or use algebraic representation of functions or situations (Example 22) • use functional relationships to discover new information about a given situation (Example 23) • use trigonometric functions on right triangles (Example 24)	*Example 19.* Which one of the following graphs represents the function $y = 2x^2$? 

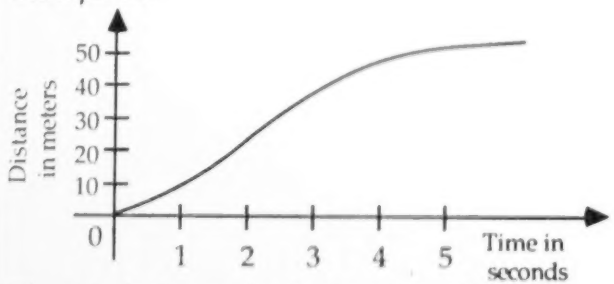
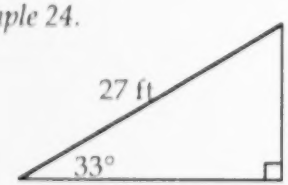
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Relations, Functions, and Graphs (cont'd.)			*Example 20.* A school bus carries 25 students. Which graph best represents the number of buses needed for any number of students up to 75? o  o  o  o 

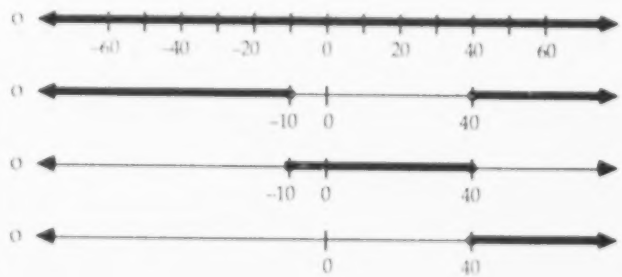
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Relations, Functions, and Graphs (cont'd.)			*Example 21.* The formula for converting degrees Celsius (C) to degrees Fahrenheit (F) is $F = \frac{9}{5}C + 32.$ To get a reasonable approximation for the temperature in degrees Celsius when the temperature is given in degrees Fahrenheit, one should: o multiply C by 2 and then add 30 o divide C by 2 and then subtract 30 o subtract 30 from F and then divide by 2 o divide F by 2 and then subtract 30 *Example 22.* The school has 1 teacher for every 24 students. Using the letter "S" for the number of students and the letter "T" for the number of teachers, write an equation expressing this relationship. o $24S = T$ o $24T = S$ o $ST = 24$ o $\frac{T}{S} = 24$

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Relations, Functions, and Graphs (cont'd.)			*Example 23.*  The graph shows the distance traveled by a moving car. During which 1-second interval will its average speed be the greatest? o 0–1 o 1–2 o 2–3 o 3–4 *Example 24.*  An escalator under construction makes an angle of 33° with the floor and is 27 feet long. Which of the following gives the distance between floors? o $x = 27 \sin 33°$ o $x = 27 \cos 33°$ o $x = 27 \tan 33°$ o $x = \frac{\sin 33°}{27}$

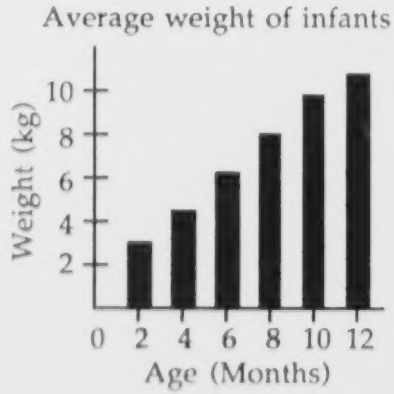
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
3. Algebra	12	The student will: • apply the relationships "greater than," "less than," and "equality" (Example 25) • use the evaluation and simplification of algebraic expressions and formulas to solve problems (Example 26) • formulate and/or solve equations, inequalities, and formulas (Example 27)	*Example 25.* Do not pour concrete when the temperature, in degrees Celsius, is less than –10 or greater than 40. According to the direction above, which of the following represents the best temperature range for pouring concrete?  *Example 26.* The number of kilometers, s, that a projectile travels in t seconds is given by the equation $s = \frac{1}{2}t^2 + 30t - k$ How far will a projectile travel in 6 seconds when k = 4? o 32 km o 185 km o 182 km o 194 km

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
3. Algebra (cont'd.)			*Example 27.* A salesperson is paid $45 per day plus $20 for every dishwasher he or she sells. If he or she wants to earn at least $400 per week (5 days), which inequality shows how many dishwashers, x, must be sold during the week? o $20x + 5(45) \geq 400$ o $5(20x + 45) \leq 400$ o $20x + 45 \geq 400$ o $20x + 45 \leq 400$

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
C. Data Organization and Interpretation/Probability			
1. Organizing Data as Graphs and Charts	16	The student will: • recognize useful ways to organize data in tables, charts, or diagrams • interpret tables or diagrams (Example 28)	*Example 28.*  According to the graph above, what was the average weight gain of infants from 6 to 10 months? o 1 kg o 2 kg o 4 kg o 6 kg

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Statistics	16	The student will: • choose and/or calculate an appropriate measure of central tendency or dispersion—mean, mode, median, midrange, range, quartile, standard deviation (Example 29) • use and interpret common statistical measures correctly (Example 30)	*Example 29.* During basketball season Margaret kept a record of the points she scored in each game. In the first seven games she scored 20, 13, 10, 14, 6, 10, and 11 points, respectively. What was her median score? o 10 o 11 o 12 o 13 *Example 30.* In the lottery game WINOT you have a 20% chance of winning. In the lottery game WINONE your chance of winning is 40%. Which of the following is always true? o You win in WINONE and lose in WINOT. o In 100 games of WINOT you will win 20 games. o You are more likely to win in WINONE than in WINOT. o You will win twice as much in WINONE as in WINOT.

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
3. Probability and Systematic Counting	16	The student will: • use empirical data to estimate probabilities of events (Example 31) • identify sample spaces representing outcomes and/or assignment of probabilities to each element in the sample space (Example 32) • form probabilities using dependent and/or independent events (Example 33) • use combinatorial reasoning, i.e., number of arrangements or selections (Example 34)	*Example 31.* The accompanying table shows the opinions of 2000 employees on a company's proposal to increase fringe benefits rather than increase wages. Total / For / Against / No opinion Male 1260 425 396 439 Female 740 208 260 272 Find the probability that a female selected is either for the proposal or of no opinion. o .130 o .568 o .260 o .698

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
3. Probability and Systematic Counting (cont'd.)			*Example 32.* If the above spinners are spun at the same time, which is the list of all possible outcomes and their probabilities?

o

event	probability
black and orange	1/6
black and green	1/6
black and yellow	1/6
white and orange	1/6
white and green	1/6
white and yellow	1/6

o

event	probability
black and white	1/2
green, orange, and yellow	1/3

o

event	probability
black and orange	1/2
white and green	1/2
orange and white	1/3
green and black	1/3
yellow and black	1/3
yellow and white	1/3

o

event	probability
black	1/5
white	1/5
orange	1/5
yellow	1/5
green	1/5

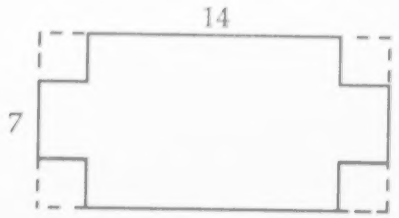
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
3. Probability and Systematic Counting (cont'd.)			*Example 33.* A car has 8 spark plugs, 2 of which are defective. Find the probability of locating both defective spark plugs in only two random selections. o $\frac{1}{64}$ o $\frac{1}{28}$ o $\frac{1}{4}$ o $\frac{1}{16}$ *Example 34.* A committee of exactly 3 persons is to be selected from 5 volunteers. How many different committees of 3 persons can be formed? o 10 o 15 o 20 o 60

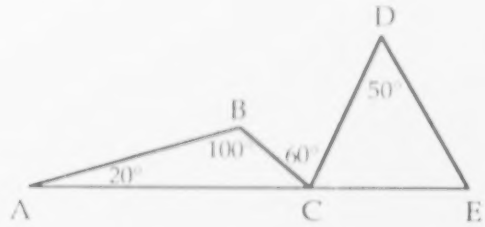
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
D. Measurement, Geometry, and Spatial Relationships			
1. Mensuration	12	The student will: • recognize a reasonable size or unit of measure in relationship to an object being measured in U.S. Customary units or metric units (Example 35) • measure one-, two-, and three-dimensional geometric figures (Examples 36, 37)	*Example 35.* Which object would be measured in liters? I. amount of water in a spa II. heaviness of a watermelon III. amount of paint needed to paint a room IV. how much lawn can be seeded from a box of grass seed o I and II o I and III o II and IV o III and IV *Example 36.* If a cube has a total surface area of 1 cm^2, then its volume is: o less than 1 cm^3 o equal to 1 cm^3 o between 1 cm^3 and 10 cm^3 o equal to or greater than 10 cm^3

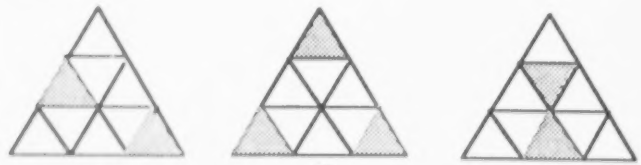
ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

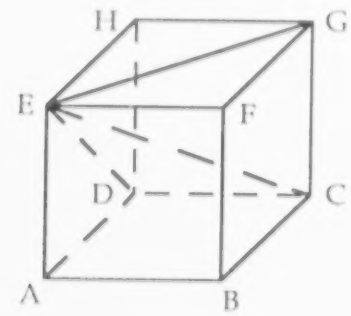
Reporting Category	Number of Items	Description of Category	Illustrative Test Items
1. Mensuration (cont'd.)			*Example 37.*  Four 2 × 2 inch squares are cut from the corners of a 7 × 14 inch rectangular piece of cardboard, which is then folded to form an open box. The volume of this box would be: o 196 cubic inches o 120 cubic inches o 98 cubic inches o 60 cubic inches

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

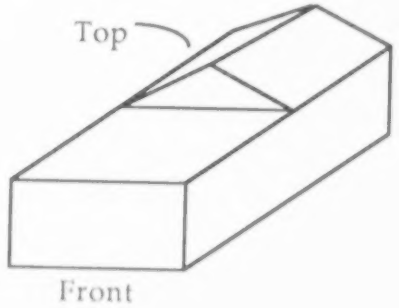
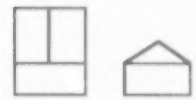
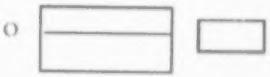
Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Geometric and Spatial Relationships	36	The student will: • use the properties of geometric figures (Examples 38, 39, 40) • identify and/or use geometric relationships, including parallelism, perpendicularity, symmetry, transformations, and the Pythagorean theorem (Example 41) • use coordinate geometry (Example 42) • use visualization of two- and three-dimensional objects (Examples 43, 44)	*Example 38.*  In the figure above, if A, C, and E lie on a line, then the degree measure of ∠ CED is: o 30 o 60 o 50 o 70 *Example 39.* For customer appeal, the diameter of a liquid soap container in the form of a cylindrical can was doubled and its height cut in half. The volume [$V = \pi r^2 h$] of the new container would be: o half of the original o equal to the original o double the original o four times larger than the original

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Geometric and Spatial Relationships (cont'd.)			*Example 40.* A girl scout troop hiked 7 km in one direction and then 12 in another. Which of the following could be the troop's distance from the starting point? o 4 km o 10 km o 19 km o 20 km *Example 41.*  I II III Which of the above figures has or have only one line of symmetry? o I only o III only o I and III o II and III

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Geometric and Spatial Relationships (cont'd.)			*Example 42.* A map carries the following directions. Start at the point G (0, 0), proceed due east to the point W (5, 0). Looking toward the northwest, you will see a lone, tall tree at the point T (0, 12). The treasure is at the point X, equidistant from W and T. What are the coordinates of X? o (0, 6) o (2.5, 0) o (2.5, 6) o (6, 2.5) *Example 43.*  If this figure represents a cube (all edges the same length), which of the following is true? o EG > ED o EG > EC o EF > DB o EG = DB

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
2. Geometric and Spatial Relationships (cont'd.)			*Example 44.*  The view directly from the top and front would be: o  o  o o

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
E. Logical Reasoning Quantifiers, Connectives, Relationships, and Using Deductive and Inductive Reasoning	12	The student will: • demonstrate knowledge of the following quantifiers: all, some, at least, at most; and connectives: and, or, not (Example 45) • recognize logical relationships, i.e., equivalence, implication, converse, inverse, and the contrapositive for a given statement (Example 46) • recognize a specific application of a general principle or of a definition (Example 47) • use diagrams to represent and solve logical problems and/or interpret diagrams (Example 48) • reason deductively and make valid inferences using conditional statements (Example 49) • use inductive reasoning to formulate conjectures and counterexamples • judge the validity of a chain of reasoning (Example 50)	*Example 45.* Which one of the following is a true statement? o $2 \cdot 3 = 6$ or $4 \cdot 2 = 6$ o $2 \cdot 3 = 6$ and $4 \cdot 2 = 6$ o If $2 \cdot 3 = 6$ then $4 \cdot 2 = 6$ o $2 \cdot 3 = 5$ or $4 \cdot 2 = 6$ *Example 46.* The statement "Lazy students do not study" is logically equivalent to: o If you are lazy, then you study. o If you are not lazy, then you study. o If you do not study, then you are lazy. o If you study, then you are not lazy.

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
Quantifiers, Connectives, Relationships, and Using Deductive and Inductive Reasoning (cont'd.)			*Example 47.* In the triangles above, x = A and y = B. I. x + A + C = 180° II. x + B + C = 180° III. A + B + x = 180° Which of the statements above is or are true? o I only o II only o III only o I, II, and III *Example 48.* In a club with 30 members, 17 take mathematics, 5 take both mathematics and biology, and 8 take neither mathematics nor biology. How many take biology but not mathematics? o 0 o 10 o 5 o cannot tell

ASSESSMENT CATEGORIES IN MATHEMATICS, *SURVEY OF ACADEMIC SKILLS: GRADE 12*

Reporting Category	Number of Items	Description of Category	Illustrative Test Items
Quantifiers, Connectives, Relationships, and Using Deductive and Inductive Reasoning (cont'd.)			*Example 49.* If N is an integer, then N is a rational number. If N is a rational number then N is a real number. What conclusion can be made from the above statements? o N is a real number. o N is an integer, rational, and a real number. o If N is an integer, then N is a real number. o If N is a real number, then N is an integer. *Example 50.* Here are some algebraic steps performed in solving the equation $2x^2 - 8x = 0$: I. $2x^2 = 8x$ II. $x^2 = 4x$ III. $x = 4$ Which is the first step, if any, that contains an error? o I o II o III o No error occurs

Answer Key

1. B	11. C	21. C	31. C	41. C
2. D	12. B	22. B	32. A	42. C
3. D	13. A	23. B	33. C	43. D
4. D	14. C	24. A	34. D	44. A
5. C	15. D	25. C	35. C	45. A
6. B	16. C	26. D	36. A	46. D
7. D	17. C	27. A	37. D	47. B
8. D	18. A	28. C	38. D	48. C
9. A	19. A	29. B	39. C	49. C
10. D	20. C	30. C	40. B	50. C

Note: Answer choices listed in two columns should be read as follows:

A	C
B	D

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0-8011-0238-3	Boating the Right Way (1985)	4.00
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0-8011-0291-X	Handbook for Teaching Pilipino-Speaking Students (1986)	4.50
0-8011-0204-9	Handbook for Teaching Portuguese-Speaking Students (1983)	4.50
0-8011-0250-2	Handbook on California Education for Language Minority Parents—Chinese/English Edition (1985)	3.25*
0-8011-0737-7	Here They Come: Ready or Not—Report of the School Readiness Task Force (1988)	2.00
0-8011-0712-1	History-Social Science Framework for California Public Schools (1988)	6.00
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